Quiz 15

March 29, 2017

Show all work and circle your final answer.

1. Determine whether the following series converge or diverge. State all tests used.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^n}$

Use root test since everything is raised to the n:

$\lim_{n\to\infty} \sqrt[n]{\left|\frac{(-2)^n}{n^n}\right|} = \lim_{n\to\infty} \frac{2}{n} = 0 < 1$, so $\boxed{\text{converges}}$ by $\boxed{\text{root test}}$.

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{2n+4^n}$

NOTE: To use ratio test, $\lim_{n\to\infty} \left|\frac{2n+4^n}{2(n+1)+4^{n+1}}\right| = \lim_{n\to\infty} \left|\frac{2n/4^n + 1}{2(n+1)/4^n + 4}\right| = \frac{1}{4}$ since $\lim_{n\to\infty} an+b/4^n = 0$ by L'H.

Use direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{4^n} = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n$, which converges by geometric series since $|r| = \left|\frac{1}{4}\right| < 1$.

$\frac{1}{2n+4^n} < \frac{1}{4^n}$ since $2n+4^n > 4^n$ when $n \geq 1$, and $\frac{1}{2n+4^n} > 0$, $\frac{1}{4^n} > 0$ when $n \geq 1$. So $\sum_{n=1}^{\infty} \frac{1}{2n+4^n}$ $\boxed{\text{converges by DCT}}$

not on quiz → (c) $\displaystyle\sum_{n=1}^{\infty} \sin\frac{1}{n^2}$

Use limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$, which converges by p-series since $2 > 1$.

$\lim_{n\to\infty} \frac{\sin\frac{1}{n^2}}{1/n^2} = 1$ since $\lim_{x\to 0} \frac{\sin x}{x} = 1$, and $\sin\frac{1}{n^2} > 0$ when $n \geq 1$ (since $0 < \frac{1}{n^2} \leq 1 < \pi$), $\frac{1}{n^2} > 0$ when $n \geq 1$.

So $\sum_{n=1}^{\infty} \sin\frac{1}{n^2}$ $\boxed{\text{converges by LCT}}$.

2. Find the radius of convergence and interval of convergence of the following series:

$$\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n}(x-3)^n$$

Use ratio test:

$$\lim_{n\to\infty} \left| \frac{\sqrt{n+1}\,(x-3)^{\cancel{n+1}}}{8^{\cancel{n+1}}} \cdot \frac{\cancel{8^n}}{\sqrt{n}\,\cancel{(x-3)^n}} \right| = \lim_{n\to\infty} \left| \frac{\sqrt{n+1}\,(x-3)}{8\sqrt{n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{\sqrt{1+1/n}\,(x-3)}{8\sqrt{n/n}} \right| \quad \left(\text{by mult. by } \frac{1/\sqrt{n}}{1/\sqrt{n}}\right)$$

$$= \left| \frac{x-3}{8} \right| \underset{\text{set}}{<} 1$$

$$-1 < \frac{x-3}{8} < 1$$

$$-8 < x-3 < 8$$

$$-5 < x < 11$$

$$\boxed{R = 8}$$

Check the endpoints of the interval $(-5, 11)$:

$x=-5$: $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n}(-8)^n = \sum_{n=1}^{\infty} (-1)^n\sqrt{n}$ diverges by test for divergence

(not by AST, which only tells us about conv.)

$x=11$: $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{8^n}(8)^n = \sum_{n=1}^{\infty} \sqrt{n}$ diverges by test for divergence

So $\boxed{I = (-5, 11)}$